radians

1. Solve $2\cos 2x - 5\cos x - 4 = 0$ for $0 \le x < 2\pi$. **5**
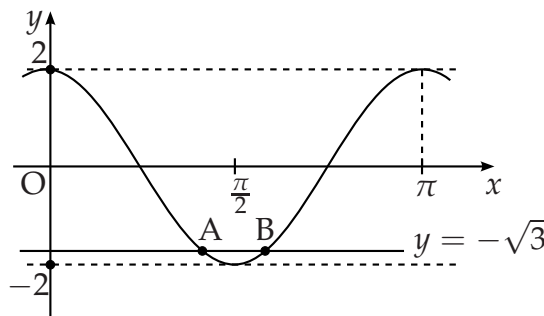
Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	5	B	CN	T10, T7	$x = 2{\cdot}419, 3{\cdot}864$	2010 P2 Q4

$\bullet^1$ ss: know to use double angle formula $\bullet^2$ ic: express as quadratic in $\cos x$ $\bullet^3$ ss: start to solve $\bullet^4$ pd: reduce to equations in $\cos x$ only $\bullet^5$ pd: complete solutions to include only one where $\cos x = k$ with $|k| > 1$	$\bullet^1$ $2 \times (2\cos^2 x - 1)\cdots$ $\bullet^2$ $4\cos^2 x - 5\cos x - 6 = 0$ $\bullet^3$ $(4\cos x + 3)(\cos x - 2) = 0$ $\bullet^4$ $\cos x = -\frac{3}{4}$ and $\cos x = 2$ $\bullet^5$ $2{\cdot}419, 3{\cdot}864$ and no solution.

2. The diagram shows the graph of a cosine function from 0 to π.

   (a) State the equation of the graph. **1**

   (b) The line with equation $y = -\sqrt{3}$ intersects this graph at point A and B.

     Find the coordinates of B. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	1	C	NC	T4	$y = 2\cos 2x$	2002 P1 Q8
(b)	3	C	NC	T7	$B(\frac{7\pi}{12}, -\sqrt{3})$	

$\bullet^1$ ic: interpret graph $\bullet^2$ ss: equate equal parts $\bullet^3$ pd: solve linear trig equation in radians $\bullet^4$ ic: interpret result	$\bullet^1$ $2\cos 2x$ $\bullet^1$ $2\cos 2x = -\sqrt{3}$ $\bullet^2$ $2x = \frac{5\pi}{6}, \frac{7\pi}{6}$ $\bullet^3$ $x = \frac{7\pi}{12}$

3.

    (*a*) Solve $\cos 2x° - 3\cos x° + 2 = 0$ for $0 \le x < 360$.   **5**

    (*b*) Hence solve $\cos 4x° - 3\cos 2x° + 2 = 0$ for $0 \le x < 360$

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	5	C	NC	T7	$x = 0, 60, 300$	2011 P1 Q23
(*b*)	2	A	NC	T11	$x = 0, 30, 150, 180, 210, 330$	

- $\bullet^1$ ss: know to use double angle formula
- $\bullet^2$ ic: express as a quadratic in $\cos x°$
- $\bullet^3$ ss: start to solve
- $\bullet^4$ pd: reduce to equations in cos only
- $\bullet^5$ ic: process solutions in given domain

- $\bullet^6$ ic: interpret relationship with (a)
- $\bullet^7$ ic: interpret periodicity

- $\bullet^1$ $2\cos^2 x° - 1 \ldots$
- $\bullet^2$ $2\cos^2 x° - 3\cos x° + 1 = 0$
- $\bullet^3$ $(2\cos x° - 1)(\cos x° - 1)$
- $\bullet^4$ $\cos x° = \frac{1}{2}, 1$
- $\bullet^5$ $0, 60, 300$

- $\bullet^6$ $2x = 0$ and 60 and 300
- $\bullet^7$ $0, 30, 150, 180, 210$ and 330

4. Find the values of t, where $0 < t < 2\pi$, for which $4\cos\left(2t - \frac{\pi}{4}\right)$ has its maximum value.   **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T7		1989 P1 Q15

- $\bullet^1$ $\cos\left(2t - \frac{\pi}{4}\right) = 1$
- $\bullet^2$ $2t - \frac{\pi}{4} = 0$
- $\bullet^3$ $t = \frac{\pi}{8}$
- $\bullet^4$ $\frac{\pi}{8}, \frac{9\pi}{8}$

5. The diagram shows part of the graph of $y = a\sin 3x°$
and the line with equation $y = \frac{2}{3}a$.
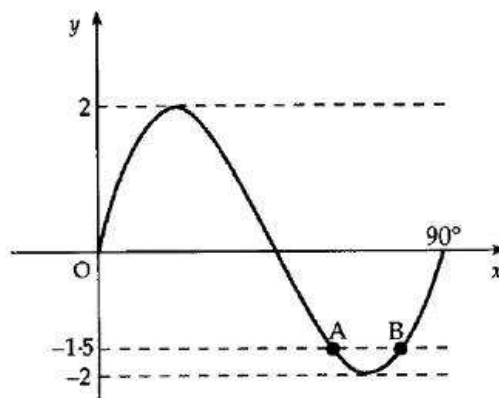Find the x-coordinates of A and B.

4
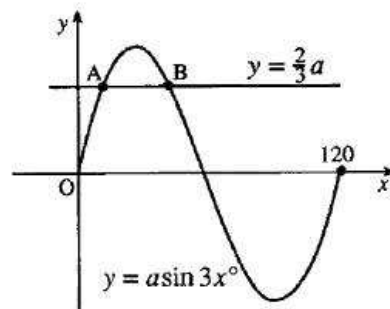
Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T7		1999 P1 Q14

•^{1} $a\sin 3x = \frac{2}{3}a$ stated or implied by •^{2}

•^{2} $\sin 3x = \frac{2}{3}$

•^{3} $3x = 41.8,\ 138.2$ (138.2 stated or implied by 46.1 in •^{4})

•^{4} $13.9,\ 46.1$

6. The diagram shows the graph of a sine function
from 0° to 90°.
(a) State the equation of the graph.
(b) The line with equation $y = -1\cdot5$ intersects the
curve at A and B.
Find the coordinates of A and B.

2

3

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	2	C	CR	T1, A7		1990 P1 Q10
(b)	3	C	CR	T7		

•^{1} $\sin 4x$

•^{2} (trig function) $\times 2$

•^{3} $f(x) = -1\cdot5$

•^{4} $57\cdot1°$

•^{5} $77\cdot9°$

7. Find, correct to one decimal place, the value of x between 180 and 270 which satisfies the equation $3\cos(2x° − 40°) − 1 = 0$.

5

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	5	C	CR	T7		1992 P1 Q5

- $\bullet^1$ $\cos(2x − 40)° = \frac{1}{2}$
- $\bullet^2$ $\cos^{-1}\frac{1}{3} = 70.53$
- $\bullet^3$ $2x − 40 = 70.5 \quad 289.5 \quad 430.5 \quad 649.5$
- $\bullet^4$ $x = 55.25 \quad 164.75 \quad 235.25 \quad 344.75$
- $\bullet^5$ $x = 235.25$

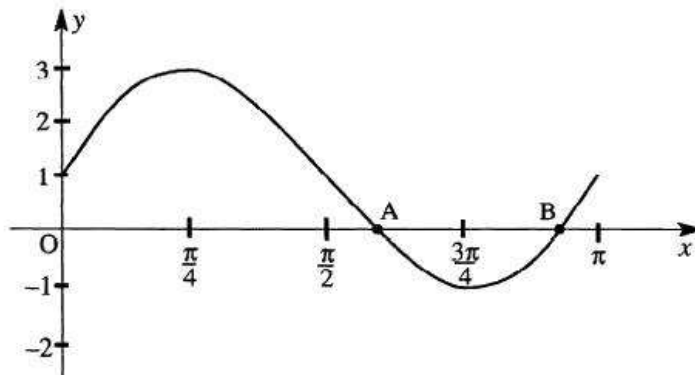8. If $f(a) = 6\sin^2 a − \cos a$, express $f(a)$ in the form $p\cos^2 a + q\cos a + r$.

Hence solve, correct to three decimal places, the equation $6\sin^2 a − \cos a = 5$ for $0 \le a \le \pi$.

4

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	2	C	CR	T7		1993 P1 Q17
	2	A/B	CR	T7		

- $\bullet^1$ subst. leading from $\sin^2$ to $\cos^2$
- $\bullet^2$ $−6\cos^2 a − \cos a + 6 = 5$
- $\bullet^3$ solving the quadratic
- $\bullet^4$ 1.231 **and** 2.094

9. The diagram below shows the graph of $y = 2\sin 2x + 1$ for $0 \le x \le \pi$.

(a) Find the coordinates of A and B (as shown in the diagram) by solving an appropriate equation algebraically. **(5)**

(b) The points $(0, 2)$ and $(\pi, 0)$ are joined by a straight line l. In how many points does l intersect the given graph? **(1)**

(c) C is the point on the given graph with an x-coordinate of $\frac{\pi}{2}$. Explain whether C is above, below or on the line l. **(3)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	3	C	NC	T7		1993 P2 Q6
(a)	2	A/B	NC	T7		
(b)	1	C	NC	CGD		
(c)	3	A/B	NC	CGD		

(a) $\bullet^1$ $2\sin 2x + 1 = 0$

  $\bullet^2$ $\sin 2x = -\frac{1}{2}$

  $\bullet^3$ for any valid sol of equ. in form $\sin ax = -\frac{b}{c}$

  $\bullet^4$ $\left(\frac{7\pi}{12}, 0\right)$

  $\bullet^5$ $\left(\frac{11\pi}{12}, 0\right)$

(b) $\bullet^6$ 3

(c) $\bullet^7$ $y_C = 1$

  $\bullet^8$ for a strategy to make a decision about C

  $\bullet^9$ for making a consistent decision about C

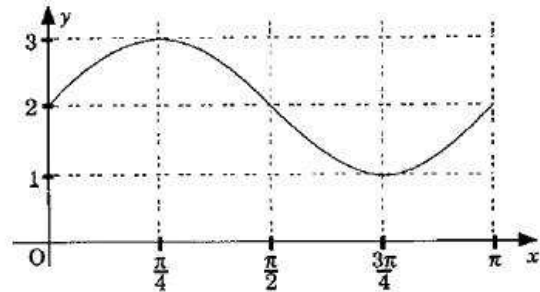10. Solve the equation $2\cos^2 x = \frac{1}{2}$, for $0 \le x \le \pi$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	3	C	NC	T7, T3		1990 P1 Q15

$\bullet^1 \quad \cos x = \pm\frac{1}{2}$

$\bullet^2 \quad x = \frac{\pi}{3}$

$\bullet^3 \quad \frac{2\pi}{3}$

11. The diagram shows the graph of the function $y = a + b\sin cx$ for $0 \le x \le \pi$.

(a) Write down the values of a, b and c. **3**

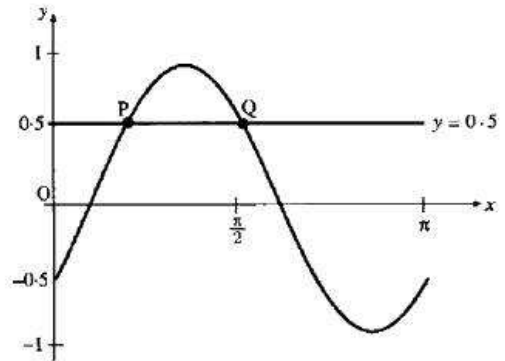(b) Find algebraically the values of x for which $y = 2.5$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	3	C	NC	T1		1994 P1 Q12
(b)	3	C	NC	T7, T3		

$\bullet^1 \quad a = 2$

$\bullet^2 \quad b = 1$

$\bullet^3 \quad c = 2$

$\bullet^4 \quad 2 + \sin 2x = 2\frac{1}{2}$

$\bullet^5 \quad 2x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\bullet^6 \quad x = \frac{\pi}{12}, \frac{5\pi}{12}$ (0.262, 1.309)

OR

$\bullet^4 \quad 2 + \sin 2x = 2\frac{1}{2}$

$\bullet^5 \quad 2x = \frac{\pi}{6}, \ x = \frac{\pi}{12}$

$\bullet^6 \quad 2x = \frac{5\pi}{6}, x = \frac{5\pi}{12}$

12. The diagram shows a sketch of the graph of

$y = \sin\left(2x - \frac{\pi}{6}\right)$, $0 \le x \le \pi$, and the straight line $y = 0.5$.

These graphs intersect at P and Q.

Find algebraically the coordinates of P and Q.

4

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T7, T3		1996 P1 Q12

$\bullet^1$ $\sin\left(2x - \frac{\pi}{6}\right) = 0.5$ *stated or implied by 2nd mark*

$\bullet^2$ $2x - \frac{\pi}{6} = \frac{\pi}{6}$

$\bullet^3$ $2x - \frac{\pi}{6} = \frac{5\pi}{6}$

$\bullet^4$ $\left(\frac{\pi}{6}, 0.5\right)$, $\left(\frac{\pi}{2}, 0.5\right)$

13. Solve the equation $2\sin\left(2x - \frac{\pi}{6}\right) = 1$, $0 \le x < 2\pi$.

4

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T7, T3		1998 P1 Q9

$\bullet^1$ $\sin\left(2x - \frac{\pi}{6}\right) = \frac{1}{2}$

$\bullet^2$ $2x - \frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}$ (*accept* 30, 150)

$\bullet^3$ $\quad x = \frac{\pi}{6}, \frac{\pi}{2}$

$\bullet^4$ $\quad x = \frac{7\pi}{6}, \frac{3\pi}{2}$

Alternative for 2nd and 3rd marks

$\bullet^2$ $2x - \frac{\pi}{6} = \frac{\pi}{6}, \quad x = \frac{\pi}{6}$

$\bullet^3$ $2x - \frac{\pi}{6} = \frac{5\pi}{6}, \quad x = \frac{\pi}{2}$

14. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

     (a) Find expressions for:

       (i) $f(h(x))$;

       (ii) $g(h(x))$.   **2**

    (b)   (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x$.

       (ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \le x \le 2\pi$.   **5**

Part	Marks	Level	Calc.	Content	Answer		U2 OC3
(a)	2	C	NC	A4	(i) $sin(x + \frac{\pi}{4})$,	(ii)	2001 P1 Q7
					$cos(x + \frac{\pi}{4})$		
(b)	5	C	NC	T8, T7	(i) proof, (ii) $x = \frac{\pi}{4}, \frac{3\pi}{4}$		

$\bullet^1$ ic: interpret composite functions	$\bullet^1$ $\sin(x + \frac{\pi}{4})$	
$\bullet^2$ ic: interpret composite functions	$\bullet^2$ $\cos(x + \frac{\pi}{4})$	
$\bullet^3$ ss: expand $\sin(x + \frac{\pi}{4})$	$\bullet^3$ $\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}$ **and**	
$\bullet^4$ ic: interpret	**complete**	
$\bullet^5$ ic: substitute	$\bullet^4$ $g(h(x)) = \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x$	
$\bullet^6$ pd: start solving process	$\bullet^5$ $(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x) - (\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x)$	
$\bullet^7$ pd: process	$\bullet^6$ $\frac{2}{\sqrt{2}} \sin x$	
	$\bullet^7$ $x = \frac{\pi}{4}, \frac{3\pi}{4}$ *accept only radians*	

[END OF QUESTIONS]